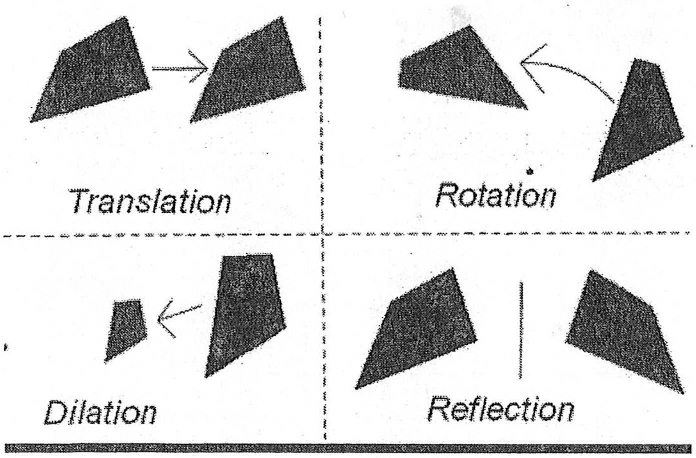
Name________________________ Notes # 33

Notes: Translations

Slide

TransfORMation:
A transformation is the change in POSITION, SIZE, or SHAPE of a figure.



4 Types of Transformations:

1) Translation
2) Rotation
3) Dilation
4) Reflection

Translation

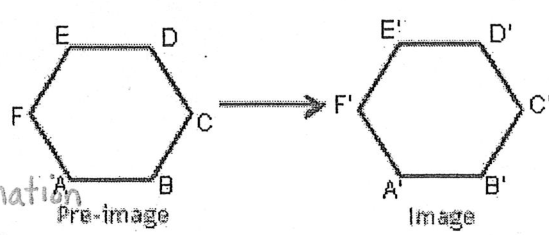
A transformation that moves each point of a figure the same distance and in the same direction.

$\langle -6,3 \rangle$
vector notation

Pre-Image = The original figure.

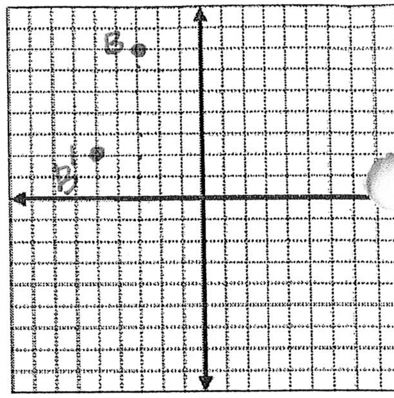
Image = Resulting (New) figure.

Pre-Image Points labeled just with letter, such as A.
Image Points labeled as A', which is read as A prime.



Isometry:
Congruent transformation
original = the new image

Example 1) Translate the point B(-3, 7) **left 2 units and down 5 units.** What are the coordinates of B'?



$B' =$ (-5, 2)

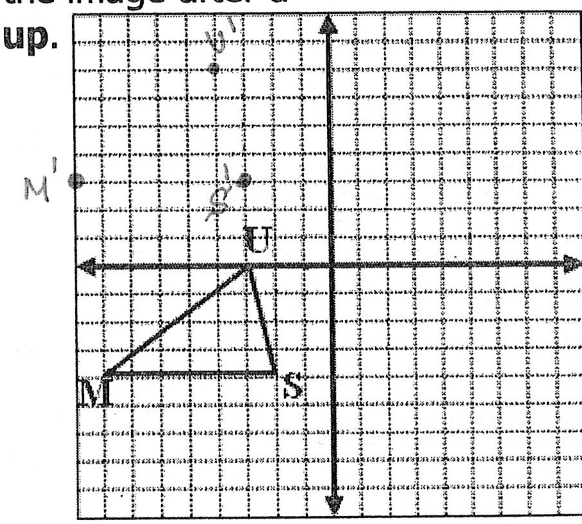
- Translating a point left and right affects the X coordinate
- Translating a point up and down affects the Y coordinate

Graph a point (x ↔, y ↕)

Vector Notation: Uses parenthesis or brackets to describe the Translation. For example, <-3,4> is a translation Left 3, Up 4.

<↔, ↕>

Example 2) ΔMSU is graphed (to the right). Graph the image after a Translation of **left 1 unit and 7 units up.**



Arrow Notation

$M(-8, -4) \rightarrow M'(-9, 3)$

$S(-2, -4) \rightarrow S'(-3, 3)$

$U(-3, 0) \rightarrow U'(-4, 7)$

$(x, y) \rightarrow (x-1, y+1)$

What is this translation in vector notation?

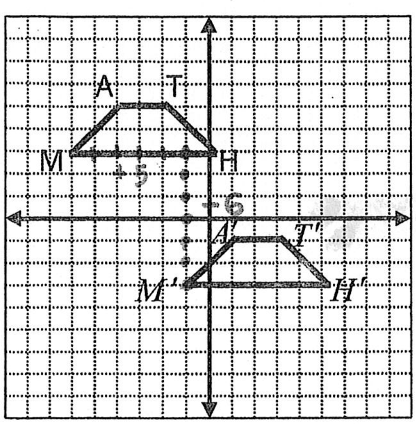
< -1 , 7 >

Example 3)

How to write a rule using arrow notation:

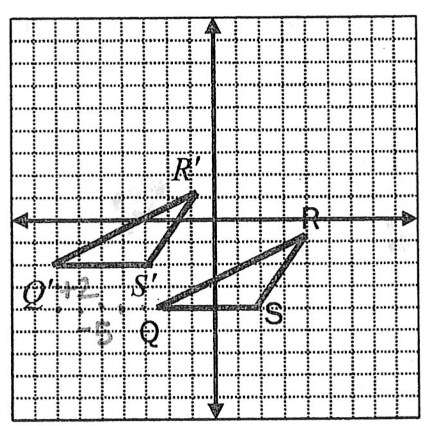
Always start with (x, y) and then describe how the X - Value moves (by adding or subtracting) and then how the Y - value moves (by adding or subtracting).

Use arrow notation to write a rule that describes the following translations.



Right 5, down 6 <5, -6>

$(x, y) \rightarrow (x+5, y-6)$



left 5, up 2 <-5, 2>

$(x, y) \rightarrow (x-5, y+2)$

Name________________________ Notes# 34

Notes: Reflections

flip

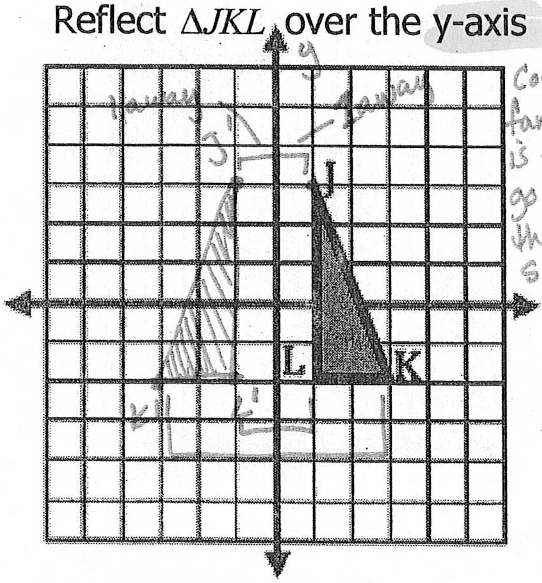
A transformation that reflects each point of the figure over a line.

Pre-Image: before reflection **Image**: After reflection

Isometry: same size

Example 1)

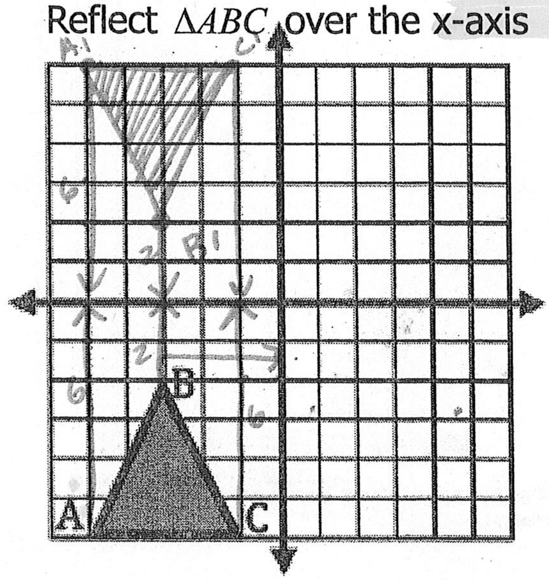
Reflect ΔJKL over the y-axis



Count how far each point is to the y-axis go that far on the opposite side

Example 2)

Reflect ΔABC over the x-axis



Reflect PQRS over the y-axis

y same x opposite

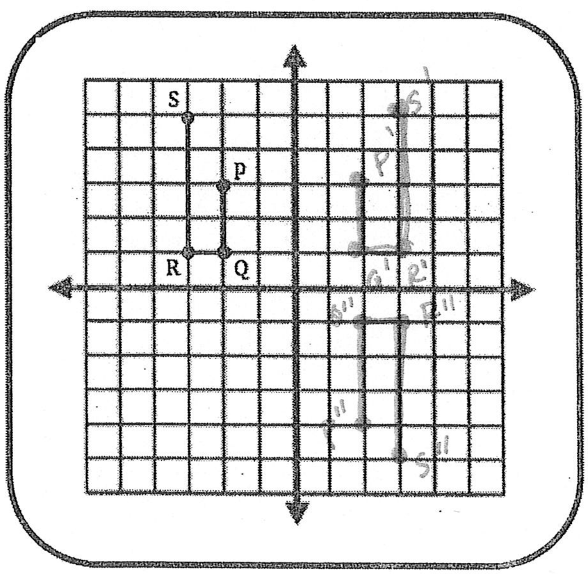
$PQRS \rightarrow P'Q'R'S'$

$P(-2,3) \rightarrow P'(2,3)$

$Q(-2,1) \rightarrow Q'(2,1)$

$R(-3,1) \rightarrow R'(3,1)$

$S(-3,5) \rightarrow S'(3,5)$



Reflect PQRS over the x-axis

x same, y opposite

$PQRS \rightarrow P''Q''R''S''$

$P(-2,3) \rightarrow P''(2,-3)$

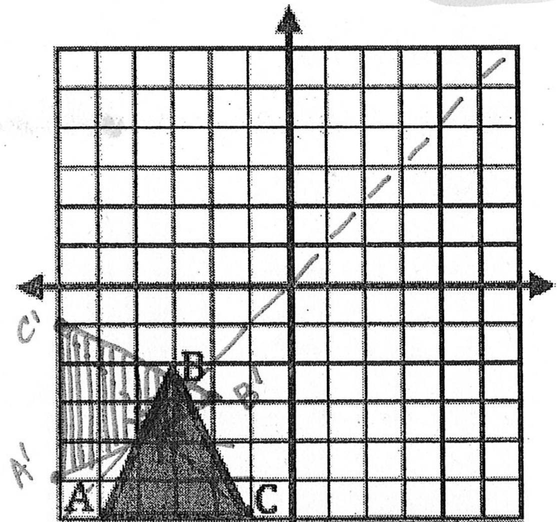
$Q(-2,1) \rightarrow Q''(2,-1)$

$R(-3,1) \rightarrow R''(3,-1)$

$S(-3,5) \rightarrow S''(3,-5)$

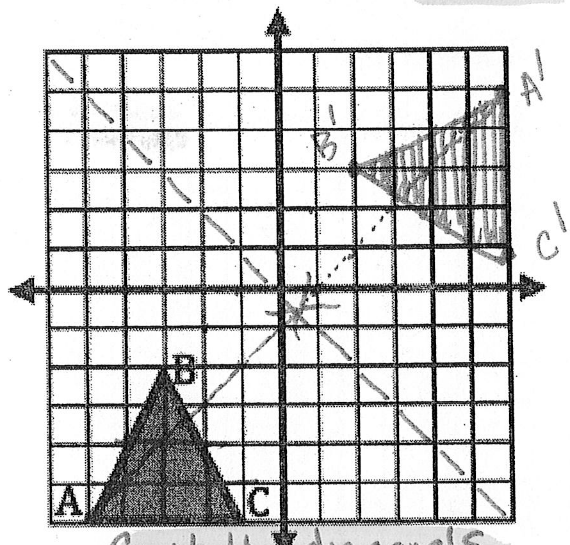
	x-axis	y-axis	both at once!
(x,y)	(x, -y)	(-x, y)	(-x, -y)

Reflection over the line y=x



count the diagonals

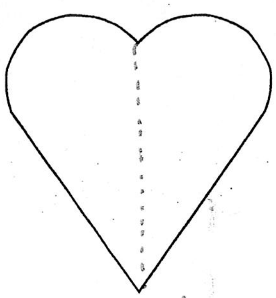
Reflection over the line y=-x



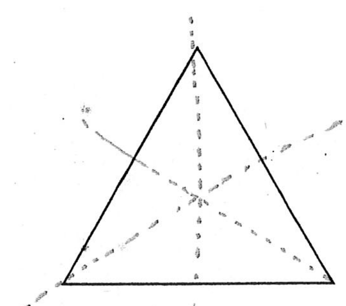
Count the diagonals

Reflectional Symmetry

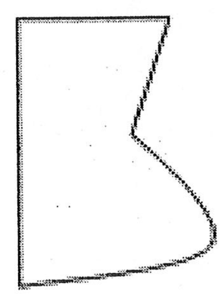
Examples: Do the following have reflectional symmetry? If so, draw the line(s) of symmetry. How many lines of symmetry does each figure have?



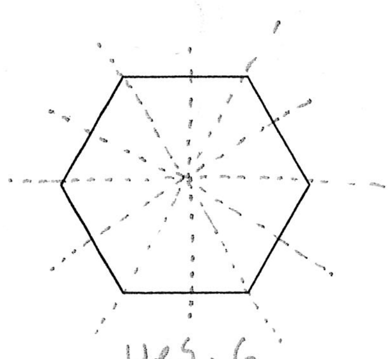
yes, 1



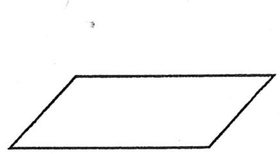
yes, 3



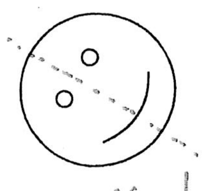
No



yes, 6



NO



yes, 1

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Notes: Rotations

otation- A rotation is a transformation that turn turns an object about a fixed (center) point

Center of Rotation- point that ________ that the figure rotates upon.

Angle of Rotation- The number of degrees a figure rotates.

**** **Positive Angle Degrees** mean to rotate a figure **COUNTERCLOCKWISE*****

Graphing Rotations:

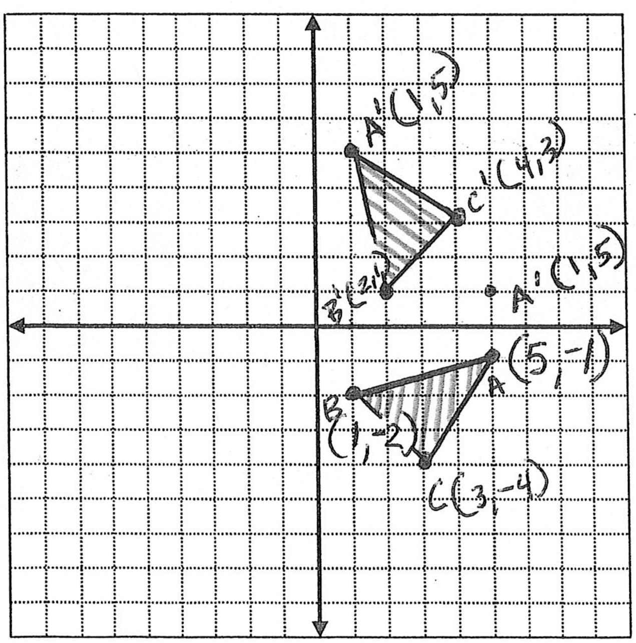
Example 1: Given a triangle with the vertices A(5, -1); B(1, -2); and C(3, -4), rotate this triangle $90°$ about the origin.

Step 1: Graph ΔABC

Step 2: Put a piece of paper over the graph. Trace ΔABC, the x-axis, the y-axis, and the origin.

Step 3: Rotate your paper counter clockwise ($90°$) about the origin. The axes should line up. Mark the position of each vertex

Step 4: Complete each figure by drawing the triangle and labeling the vertices.



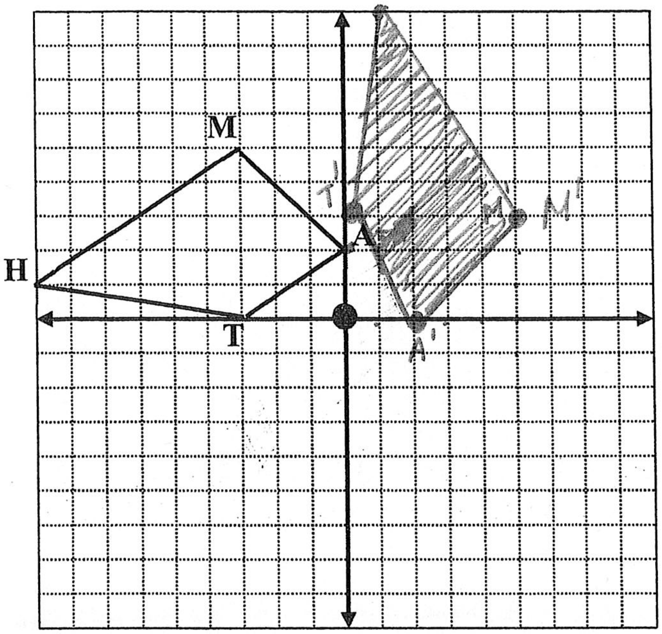
Step #5: Write the coordinates of the image.

A'(1,5)

B'(2,1)

C'(4,3)

Example 2: Rotate the following figure $270°$ around the given point.



m(⁻3,5) m'(⁺5,⁺3)

A(0,2) A'(2,0)

T(⁻3,0) T'(0,3)

H(-9,1) H'(1,9)

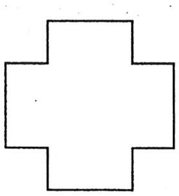
Rotational Symmetry- A figure has rotational symmetry if it can be rotated

180° degrees or less ~~and~~ onto itself ~~the original figure~~.

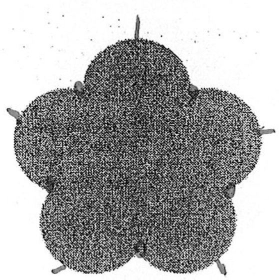
Angle of rotation = 360°/(number of symmetrical points)

Examples: Do the following figures have rotational symmetry? If so, what is their angle of rotation?

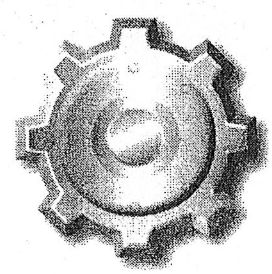
a.



b.



c.



360 ÷ 4 = 90°

360 ÷ 5 = 72°

360 ÷ 8 = 45°

Name Key Day#________

Review: Rotations

Point *O* is the center of regular hexagon *BCDEFG*. Find the image of the given point or segment for the given rotation.

7. 120° rotation of *F* about *O*. D

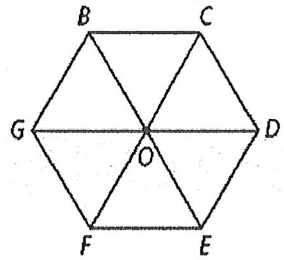
8. 180° rotation of *B* about *O*. E

9. 300° rotation of $\overline{BG}$ about *O*. $\overline{BC}$ or $\overline{CB}$

10. 360° rotation of $\overline{CD}$ about *O*. $\overline{CD}$

11. 60° rotation of *E* about *O*. D

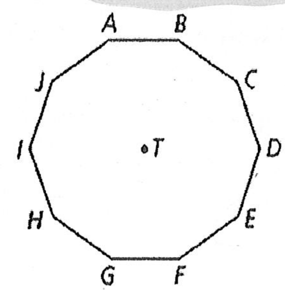
12. 240° rotation of $\overline{FE}$ about *O*. $\overline{BG}$



Angle of Rotation $= \frac{360}{n} = \frac{360}{6}$

1 turn $= 60°$
2 $= 120°$
3 $= 180°$
4 $= 240°$
5 $= 300°$
6 $= 360°$

Use the figure at the right for Exercises 13–15. Point *T* is the center of the regular decagon.



$\frac{360}{10} = 36°$

13. What is the angle of rotation that maps *D* to *A*? $3 \times 36 = 108$

14. What is the angle of rotation that maps *B* to *H*? $4 \times 36 = 144$

15. What is the angle of rotation that maps $\overline{FG}$ to $\overline{DE}$? 72°

16. Describe a composition of rotations that maps *A* to *E*. $6 \times 36 = 216$

20. A pie is cut into 12 equal slices. What is the angle of rotation about the center that will map a piece of pie to a piece that is two slices away from it? $\frac{360}{12} = 30° \times 2 = 60°$

21. ΔRST has vertices at *R*(0, 3), *S*(4, 0), and *T*(0, 0). Find the coordinates of the vertices after a 90° clockwise rotation about *T*.

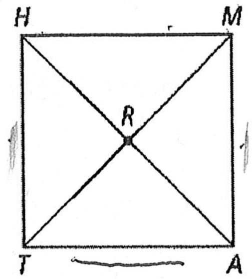
$T(0,0) \rightarrow T'(0,0)$
$R(0,3) \rightarrow R'(3,0)$
$S(4,0) \rightarrow S'(0,-4)$

22. ΔFGH has vertices *F*(−1, 2), *G*(0, 0), and *H*(3, −1). Find the coordinates of the vertices after a 90° rotation about *G*.

$(-y, x)$

$G(0,0) \rightarrow G'(0,0)$
$H(3,-1) \rightarrow H'(1,3)$
$F(-1,2) \rightarrow (-2,-1)$

Point R is the center of regular quadrilateral $MATH$. Find the image of the given point or segment for the given rotation.



$\frac{360}{4} = 90°$

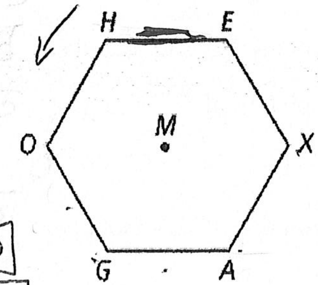
6. 90° rotation of H about R T

7. 180° rotation of M about R T

8. 270° rotation of $\overline{AT}$ about R $\overline{TH}$ or $\overline{HT}$

9. 360° rotation of $\overline{HM}$ about R $\overline{HM}$ or $\overline{MH}$

Use the figure at the right for Exercises 10–13. Point M is the center of the regular hexagon.



$\frac{360}{6} = 60°$ per turn

10. What is the angle of rotation that maps H to X? $4 \times 60 = \boxed{240°}$

11. What is the angle of rotation that maps E to G? $3 \times 60 = \boxed{180°}$

12. What is the angle of rotation that maps $\overline{HE}$ to $\overline{AG}$? $3 \times 60 = \boxed{180}$

13. What is the angle of rotation that maps $\overline{HE}$ to $\overline{GO}$? $2 \times 60 = \boxed{120}$

18. ΔXYZ has vertices at $X(2, 0)$, $Y(0, 0)$, and $Z(0, 5)$. Find the coordinates of the vertices after a 180° rotation about Y.

$(-x, -y)$

$X'(-2, 0)$
$Z'(0, -5)$
$Y'(0, 0)$

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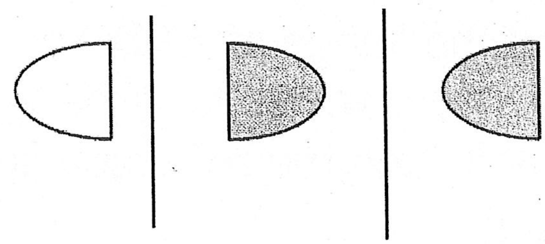
Notes: Compositions

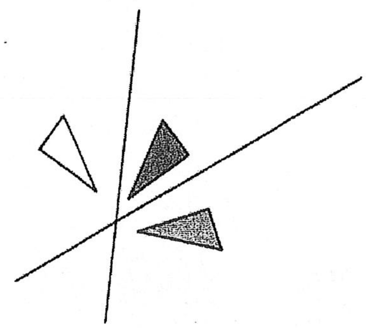
A **composition** of transformations is one transformation followed by another.

Theorem: A translation or rotation is a composition of two reflections.

Theorem: A composition of reflections over two parallel lines is a translation.



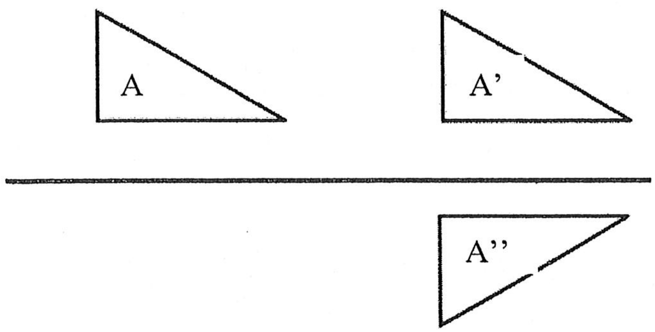
Theorem: A composition of reflections over two intersecting lines is a rotation.



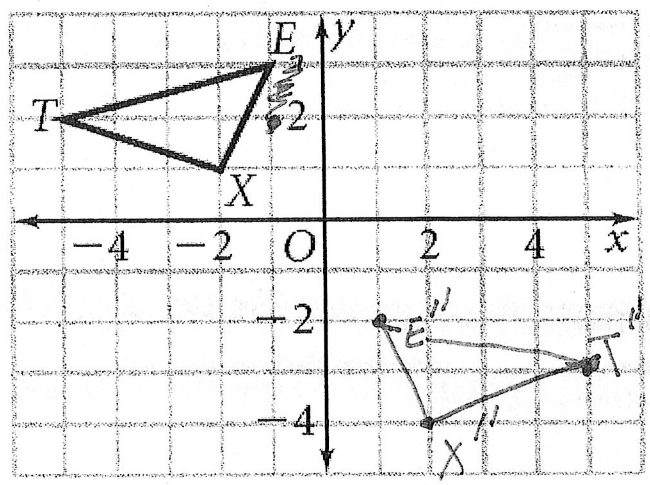
Theorem: One of two congruent figures can be mapped onto the other by a composition of at most three reflections.

Theorem: There are only four isometries. They are reflection, translation, rotation, and glide reflection.

Glide Reflection (translation followed by reflection)

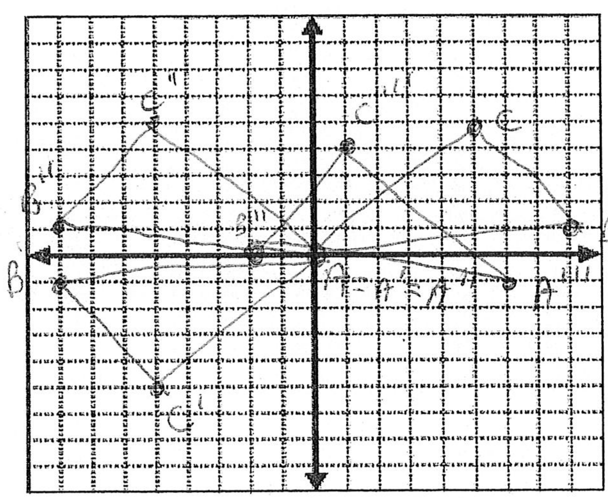


Find the image of ΔTEX for a glide reflection where the glide vector is $\langle 0, -5 \rangle$ and the reflection line is x = 0.



Pre-image: A(0,0), B(8,1), C(5,5)

Rotate the figure 180°	(−x, −y) A'(0,0) B'(−8,−1) C'(−5,−5)
Reflect the figure over the x-axis	(x, −y) A''(0,0) B''(−8,1) C''(−5,5)
Translate the figure according to (x,y)→(x+6,y-1)	A'''(6,−1) B'''(−2,0) C'''(1,4)



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Notes: Dilations

Dilation- Transformations where the preimage and image are **similar**.

There are two types:

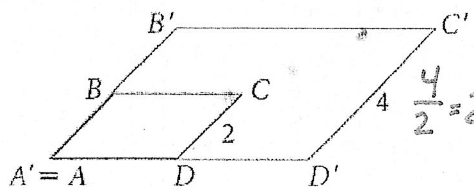
1) **Reductions**: Scale factor is between 0 and 1 } $1 < X < 0$ | Greater than Zero less than 1

common $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{3}{4}$

2) **Enlargements:** Scale factor is between 1 and ∞ } $1 < X$ | Greater than 1

Scale Factor = $\frac{\text{New}}{\text{Original}}$

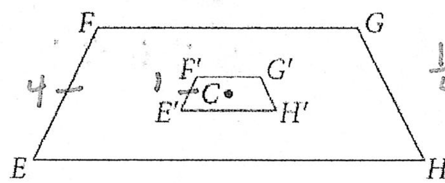
Examples:



$\frac{4}{2} = 2$

Enlargement
Center A, scale factor 2

4

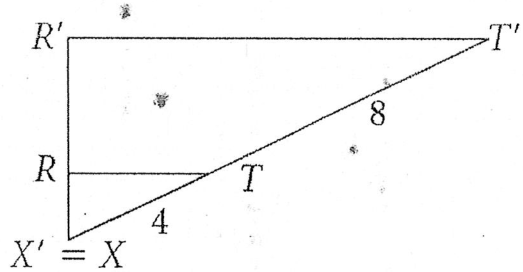


$\frac{1}{4}$

Reduction
Center C, scale factor $\frac{1}{4}$

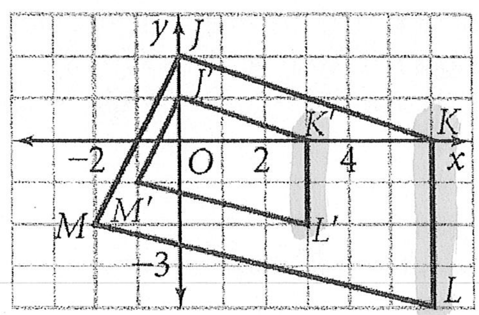
Describe the dilation. Find the center and the scale factor of the dilation.

A.



$\frac{12}{4} = 3$

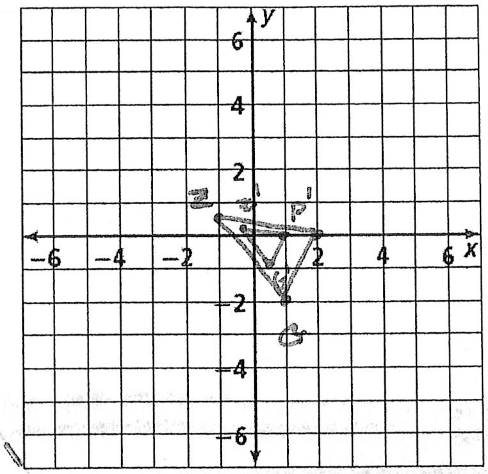
scale factor



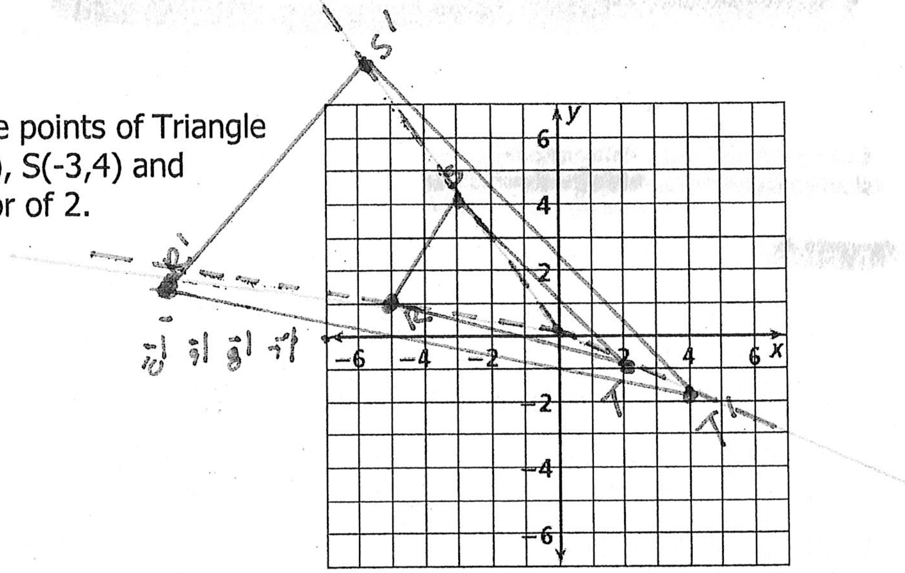
$\frac{2}{4} = \frac{1}{2}$

ΔPZG has vertices P(2, 0), Z(− 1, ½) and G(1, − 2). Use multiplication to find the image of ΔPZG for a dilation with center (0, 0) and scale factor ½. Draw the image and state the image vertices.

P' (1, 0)
Z' ($-\frac{1}{2}$, $\frac{1}{4}$)
G' ($\frac{1}{2}$, -1)



Graph and find the image points of Triangle RST with vertices R(-5,1), S(-3,4) and T(2,-1) with a scale factor of 2.

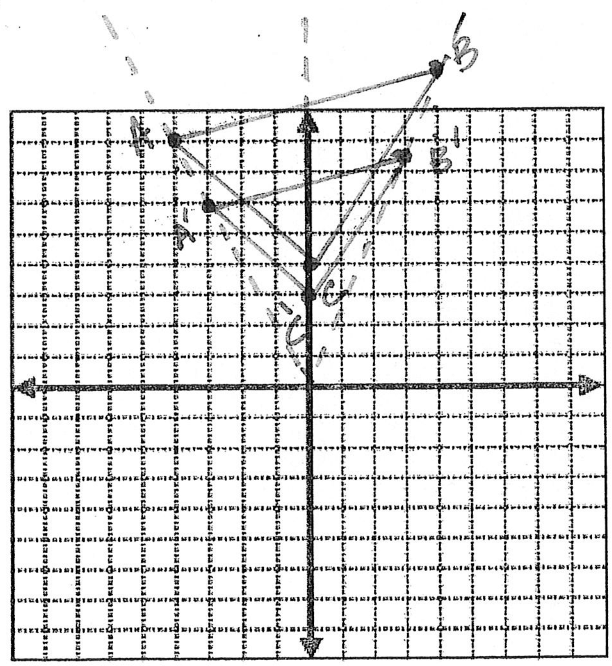


R' (-10 , 2)

S' (-6 , 8)

T' (4 , -2)

Graph and find the image points of Triangle ABC with vertices A(-4,8), B(4,10) and C(0,4) with a scale factor of 3/4.



A' (-3 , 6)

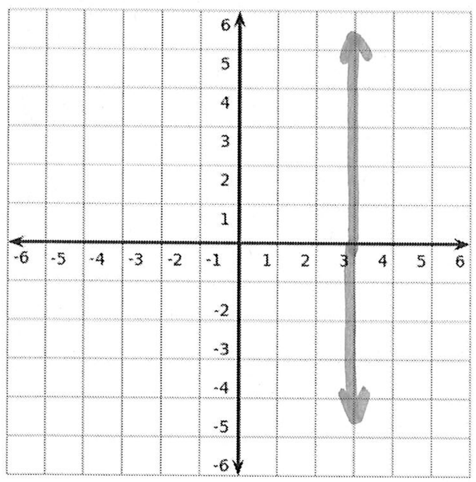
B' (3 , 7.5)

C' (0 , 3)

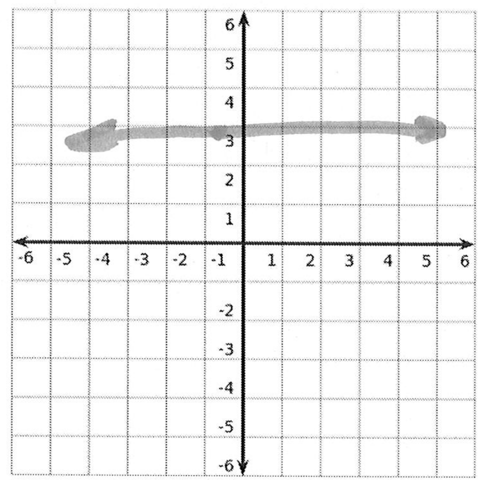
Name Key Day#________

Practice: Graphing Vertical & Horizontal Lines

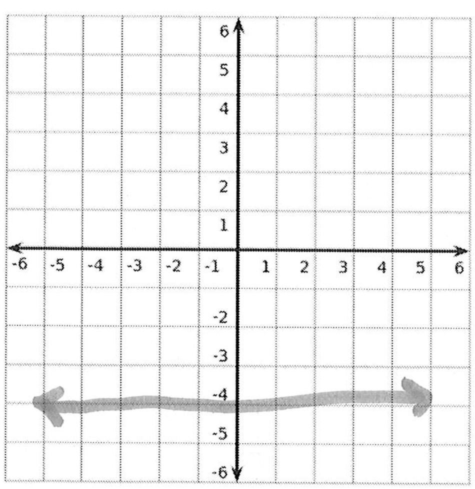
1. X= 3



2. Y= 3



3. Y= -4



4. X = -5